Discrete-Valued Vector Reconstruction by Optimization with Sum of Sparse Regularizers

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Abstract

We propose a possibly nonconvex optimization problem to reconstruct a discrete-valued vector from its underdetermined linear measurements. The proposed sum of sparse regularizers (SSR) optimization uses the sum of sparse regularizers as a regularizer for the discrete-valued vector. We also propose two proximal splitting algorithms for the SSR optimization problem on the basis of alternating direction method of multipliers (ADMM) and primal-dual splitting (PDS). The ADMM based algorithm can achieve faster convergence, whereas the PDS based algorithm does not require the computation of any inverse matrix.

1. Discrete-Valued Vector Reconstruction

**Goal:** Reconstruct a discrete-valued vector $x \in \{r_1, \ldots, r_L\}^N$ (distribution: $p_x = \Pr(x_n = r_\ell) \quad (\ell = 1, \ldots, L)$) from underdetermined linear measurements $y = Ax + v \in \mathbb{R}^M$ ($M < N$)

\[
\begin{align*}
\min_{x \in \mathbb{R}^N} & \quad \sum_{\ell=1}^L q_\ell h \left( s - r_\ell, 1 \right) + \frac{\lambda}{2} \|y - Ax\|_2^2 \\
\text{s.t.} & \quad q_\ell \geq 0, \quad \lambda > 0 : \text{parameters}
\end{align*}
\]

- faster convergence, require matrix inversion

- slower convergence, no matrix inversion

✓ The proposed approach can use any proximable sparse regularizer for $h(\cdot)$.
✓ The proposed approach can also be applied to the reconstruction of complex discrete-valued vector.

2. Proposed Method

**Sum of sparse regularizers (SSR) optimization**

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The proposed approach can also be applied to the reconstruction of complex discrete-valued vector.

Example: Binary vector reconstruction ($x \in \{-1,1\}^N$)

- $(r_1, r_2) = (-1, 1)$
- $(p_1, p_2) = (1/2, 1/2)$
- $A$: i.i.d. Gaussian

- sparse regularizer $h(\cdot)$
  - $\ell_2$ norm
  - $\ell_2/2$ norm
  - $\ell_1/2$ norm
  - $\ell_0$ norm
  - $\ell_1 - \ell_1$ difference [1]

ADMM-SSR converges faster than PDS-SSR

The proposed algorithms with nonconvex regularizers, especially with the $\ell_p$ and $\ell_0$ norms, can achieve much better SER performance.

3. Simulation Results

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