Discreteness-Aware AMP for Reconstruction of Symmetrically Distributed Discrete Variables

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Abstract

We propose a message passing-based algorithm to **reconstruct a discrete-valued vector** whose elements have a symmetric probability distribution. The proposed algorithm, referred to as **discreteness-aware approximate message passing (DAMP)**, borrows the idea of the AMP algorithm for compressed sensing. We analytically evaluate the performance of DAMP via state evolution framework to **derive a required number of linear measurements** for the exact reconstruction with DAMP.

1. Introduction

**Discreteness-aware AMP for Reconstruction of Discrete-valued Vector**

reconstruct a discrete-valued vector $b \in \mathbb{R}^N$ from its linear measurements $y = Ab \in \mathbb{R}^M$ ($N \geq M$)

**Potential applications**

- multiuser detection (Machine-to-Machine)
- MIMO signal detection (Multi-input Multiple-output)
- FTN signaling (Faster-than-Nyquist)

**Purpose of this work**

- propose a low-complexity algorithm for the discrete-valued vector reconstruction
- theoretically analyze the performance of the proposed algorithm via state evolution [1]

2. Proposed DAMP Algorithm

**Assumption**

$b \in \{0, \pm r_1, \ldots, \pm r_L\}^N$ (Pr($b_j = r_i$) = Pr($b_j = -r_i$))

$A$ is composed of i.i.d. variables with zero mean and variance $1/M$

**SOAV (Sum-of-Absolute-Value) optimization** [2]

use the fact that some elements of $b \pm r_1$ are 0

$$
\hat{b} = \arg \min_{x \in \mathbb{R}^N} \left\{ q_0 ||x||_1 + \sum_{i=1}^L q_i(||x - r_i 1||_1 + ||x + r_i 1||_1) \right\},
$$

subject to $y = Ax$

apply the idea of AMP algorithm [1]

**Proposed algorithm (DAMP: Discreteness-aware AMP)**

1. **Initialization** $x^{-1} = x^0 = 0$, $z^{-1} = 0$

2. For $t = 0, 1, \ldots$ calculate

   estimate of $b$

   $x^{t+1} = \eta(A^T z^t + x^t, \lambda \sigma_t),$

   $$
   z^{t+1} = y - Ax^{t+1} + \frac{1}{2} z^{t-1} \left( \eta'(A^T z^{t-1} + x^{t-1}, \lambda \sigma_{t-1}) \right).
   $$

   complexity: $O(MN)$ per iteration

   $$
   \Delta = M/N : \text{observation ratio}
   $$

   $\langle \cdot \rangle : \text{mean}
   $$

   $\sigma_t^2 = \frac{1}{2} ||x^t - b||_2^2 : \text{Mean-Square-Error (MSE)}$

3. State Evolution for DAMP

**State evolution** [1]

 predict the behavior of the MSE \( \{\sigma_t^2\}_{t=0,1,\ldots} \)

in the large system limit \( N, M \to \infty, M/N = \Delta \)

$$
\frac{d\Psi}{d(\sigma^2)}|_{\sigma^2=0} < 1 \Rightarrow \text{Success}
$$

**Phase transition of DAMP**

required observation ratio for DAMP

**4. Simulation Results**

$b \in \{0, \pm 1\}^N$

**empirical (N = 100)**

$P_0 = 0.2$

$\Delta = 0.7$

$b \in \{0, \pm 1\}^N$

$N = 1000$

approach the theoretical performance as $N$ increases

rapidly increase around the theoretical boundary
